Closing Wed: HW_9A, 9B (9.3, 9.4)
Final Exam, Saturday, March 10^{th}
Kane 210, 1:30-4:20pm

9.4 Diff. Eq. Apps *(continued)*

Entry Task: Assume a 12 Liter vat contains 7 kg of salt initially. A pipe pumps in *pure water* at 3 L/min. The vat is well mixed. The mixture drains at 3 L/min.
Let $y(t)$ = "kg of salt in vat at time t".

Identify and label the following:

1. Volume of the vat (Is it changing?)
2. Amount of salt per min entering.
3. Amount of salt per min exiting.
4. Initial amount of salt.

Example: Assume a 12 Liter vat contains 7 kg of salt initially. A pipe pumps in salt water (brine) at 3 L/min with a concentration of 2 kg/L of salt. The vat is well mixed. The mixture drains at 3 L/min.

Let $y(t)$ = "kg of salt in vat at time t".

(a) Find $y(t)$.

(b) Find the limit of $y(t)$ as $n \to \infty$.

Mixing Problem Summary

V = volume of vat (liters)

t = time (min)

$y(t)$ = amount in vat (kg)

$\frac{dy}{dt}$ = rate (kg/min)

$$\frac{dy}{dt} = \text{Rate In } - \text{Rate out}$$

$$= \left(?\frac{kg}{L}\right)\left(?\frac{L}{min}\right) - \left(\frac{y}{V}\ \frac{kg}{L}\right)\left(?\frac{L}{min}\right)$$

$$y(0) = ?\text{ kg}$$

Example: Assume a 100 Liter vat contains 5kg of salt initially. Two pipes (A & B) pump in salt water (brine).

Pipe A: Enters at 3L/min with a concentration of 4kg/L of salt.

Pipe B: Enters at 5L/min with a concentration of 2kg/L of salt.

The vat is well mixed.

The mixture leaves the vat at 8L/min.

Let $y(t)$ = "kg of salt in vat at time t".

How would you set this up?

Example: Assume a 50 Liter container currently has 20 Liters of water with 24 kg of dissolved salt.
A pipe pumps in *pure water* at 6 L/min.
The vat is well mixed.
The mixture drains at 4 L/min.
Let $y(t)$ = “kg of salt in vat at time t”.
What is different about this problem?

4. Air Resistance:

A skydiver steps out of a plane that is 4,000 meters high with an initial downward velocity of 0 m/s.
The skydiver has a mass of 60 kg.
(Treat downward as positive).
Let $y(t)$ = "height at time *t*"

Newton's 2nd Law says:

(mass)(acceleration) = Force

$$m\frac{d^2y}{dt^2} = \text{sum of forces on the object}$$

The force due to gravity has constant magnitude (acting downward):
$F_g = mg = 60 \cdot 9.8 = 588$ N

One model for air resistance

The force due to air resistance (*drag force*) is proportional to velocity and in the opposite direction of velocity. So

$$F_d = -k\,v \quad \text{Newtons}$$

Assume for this problem k = 12.

Spring 2011 Final:

$v(t)$ = velocity of an object

$$F = mg - kv$$

Recall:

$$F = ma = m\frac{dv}{dt}$$

You are given m, g, and k and asked for solve for $v(t)$.

Spring 2014 Final:
A lake has a volume of 1000 cubic meters and contains pure water. A farm near the lake begins using a pesticide. Runoff from the farmland into the lake is 10 cubic meters of water per day, with a concentration of 50 grams of pesticide per cubic meter of water. The lake drains to the ocean at a rate of 10 cubic meters per day.

Find the formula *p(t)* for the amount of pesticide in the late at time *t* days.

Winter 2011 Final:

Your friend wins the lottery, and gives you P_0 dollars to help you pay your college expenses. The money is invested in a savings account that earns 10% annual interest, compounded continuously, and you withdraw the money continuously at an average rate of $3600 per year.

Find the formula *A(t)* for the amount of money in the account after *t* years.

Fall 2009 Final:

The swine flu epidemic has been modeled by the Gompertz function, which is a solution of

$$\frac{dy}{dt} = 1.2\, y\, (K - ln(y)),$$

where $y(t)$ is the number of individuals (in thousands) in a large city that have been infected by time t, and K is a constant.

On July 9, 2009, 75 thousand individuals had been infected.

One month later, 190 thousand individuals had been infected.

Find the formula *y(t)* for the number of people that are infected *t* months, July 9, 2009.

Side Note on Population Modeling

The Logistics Equation

Consider a population scenario where there is a limit (capacity) to the size of the population.

Let $P(t)$ = population size at time t.

M = maximum population size. (capacity)

We sometimes want a model that

a. …is like natural growth when P(t) is significantly smaller than M;
b. …levels off (with a slope approaching zero), then the population approaches M.

One such model is the so-called logistics equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right) \text{ with } P(0) = P_0$$